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Student Mathematical Notes

Edited by JOAN L. LEVINE

Material for this department should be sent to Dr. Joan L. Levine, Department of Mathematics, Newark State College, Union, New Jersey 07083 and should include your full name, grade, school, school mailing address, and home mailing address.

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Editor. Thomas J. Hill, Department of Mathematics, University of Oklahoma, Norman, Oklahoma 73069. General editorial correspondence should be sent to him.

Associate Editor. Elroy J. Bolduc, Jr., College of Education, University of Florida, Gainesville, Florida 32601. Solutions to problems and proposed problems should be sent to him.

Associate Editor. Joan L. Levine, Department of Mathematics, Newark State College, Union, New Jersey 07083. Letters from students and student papers should be sent to her.

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Theorem of Koutsoures

The article below contains the written work of a theorem which I have discovered. I have named it after myself since I am proud to have discovered it, and I think it shows that in today's world of mathematics there are still many theorems to be sought out.

Theorem. Given n consecutive integers, the sum of these integers and n^2 is equal to the sum of the next n consecutive integers.

Example.

Take the four integers 7, 8, 9, 10. Here $n = 4$.

$7 + 8 + 9 + 10 + 4^2 = 50.$

And the sum of the next four consecutive integers $11 + 12 + 13 + 14$ is also 50.

Proof.

$$3^2 + x + (x + 1) + (x + 2) = (x + 3) + (x + 4) + (x + 5)$$
$$3^2 + 3x + 3 = 3x + 12.$$

$$4^2 + x + (x + 1) + (x + 2) + (x + 3) = (x + 4) + (x + 5) + (x + 6) + (x + 7)$$
$$4^2 + 4x + 6 = 4x + 22$$

The left sides of the above special cases suggest considering in general $n^2 + nx + (1 + 2 + 3 + \cdots + n - 1)$, or $n^2 + nx + \frac{(n-1)n}{2}$.

Similarly, the right sides suggest the general expression $nx + (n + n + 1 + n + 2 + \cdots + 2n - 1)$, or $nx + \frac{n(3n-1)}{2}$.

However it is an easy exercise in algebra to show that the results, on the left and on the right, are equal:

$$n^2 + nx + \frac{(n-1)n}{2} = nx + \frac{n(3n-1)}{2}.$$

On both sides, nx is the same. So we need to show that $n^2 + \frac{(n-1)n}{2}$ is the same as $\frac{n(3n-1)}{2}$.

(Continued from page 5, column 3)

$$n^2 + \frac{(n-1)n}{2} = \frac{2n^2 + n^2 - n}{2} = \frac{3n^2 - n}{2} = \frac{n(3n-1)}{2}$$

Thus, the theorem is proved.

In this proof, n can be any positive integer, and x can be any integer at all.

JIM T. KOUTSOURES
Grade 10
Maine East HS
Morton Grove, Illinois

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Editor. Elroy J. Bolduc, Jr., College of Education, University of Florida, Gainesville, Florida 32601. General editorial correspondence should be sent to him.

Associate Editor. Steven R. Conrad, Mathematics Department, Benjamin N. Cardozo High School, 57-00 223 St., Bayside, N. Y. 11364. Solutions to problems and proposed problems should be sent to him.

Associate Editor. Betty Plunkett Lichtenberg, Dept. of Mathematics Education, University of South Florida, Tampa, Florida 33620. Letters from students and student papers should be sent to her.

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Koutsoures-Baker Theorem

In the November, 1971, issue of the *Mathematics Student Journal*, there is an interesting article by Jim T. Koutsoures, a tenth grade student at Maine East High School in Morton Grove, Illinois. This young man has developed his own theorem, which he calls the Theorem of Koutsoures. The theorem is stated as follows:

Theorem: Given n consecutive integers, the sum of these integers and n^2 is equal to the sum of the next n consecutive integers.

An illustration of how the theorem may be applied is given, and a proof has also been presented. It is indeed gratifying to note that a student has discovered an important mathematical principle on his own. The present author has found a generalization of the Theorem of Koutsoures, which deals with the concept of progressions. This theorem will be referred to as the Koutsoures-Baker Theorem, and it may be stated as follows:

Theorem: If the common difference of an arithmetic progression is denoted by d, then the sum of the first k terms of the progression, increased by dk^2, is equal to the sum of the next k terms of the progression.

Example: Let $d=4$ and $k=5$. Then $dk^2=100$ and,

$$24+28+32+36+40+(100) = 44+48+52+56+60.$$

Proof of Theorem: Let the arithmetic progression be denoted by:

$$a+(a+d)+(a+2d)+\dots+[a+(k-1)d].$$

The sum of these terms is

$$\frac{k}{2}[2a+(k-1)d]. \qquad (1)$$

The next k terms of this progression are:

$$[a+kd] + [a+(k+1)d] + [a+(k+2)d] + \dots + [(a+kd)+(k-1)d].$$

The sum of these terms is

$$\frac{k}{2}[2a+2kd+(k-1)d]. \qquad (2)$$

A comparison of (1) and (2) produces the following identity:

$$\frac{k}{2}[2a+(k-1)d] + k^2d = \frac{k}{2}[2a+2kd+(k-1)d].$$

which completes the proof.

by DR. BETTY L. BAKER
Hubbard High School
Chicago, Illinois